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# Transport of carbon dioxide in linear low-density polyethylene determined by permeation measurements and NMR spectroscopy

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#### Abstract

This work reports the transport of carbon dioxide in compression molded linear low-density polyethylene films. A common characteristic of the transport is that the curves that depict the dependence of the permeability coefficient of  $CO<sub>2</sub>$  on the upstream pressure exhibit a sharp increase in the low pressure region as the upstream pressure decreases at moderately high temperatures. This behavior can be explained by assuming that absorption and adsorption processes govern the transport of  $CO<sub>2</sub>$  in the films. The dual mode parameters calculated for coextruded and compressed films are compared. The diffusion coefficient of  $CO<sub>2</sub>$  in the films is also obtained using NMR techniques. The value of the diffusion coefficient is nearly one order of magnitude higher than that determined from permeation measurements. The possible reasons for this discrepancy are discussed.  $© 2001$  Elsevier Science Ltd. All rights reserved.

Keywords: Polyethylene films; NMR spectroscopy; Permeation measurements

## 1. Introduction

Gas transport in films involves solution of the gas in the film, diffusion through it, and desorption of the gas on the other face of the film. Micro-Brownian motions of the chains in rubbery materials may provide the source of the penetrant-scale hole needed for diffusion. In most cases, the performance of films in the rubbery state is excellent as far as their permeability is concerned, but it is poor with regard to their selectivity characteristics. For this reason, glassy instead of rubbery membranes are used for separation processes.

The presence of crystalline entities in rubbery films such as poly(ethylene) may alter the permeability of the films as a consequence of an increase in the tortuosity of the diffusive path caused by the crystallites and/or a decrease of solubility in the crystalline regions. The crystallites also reduce the mobility of the chains close to the anchoring points in the crystals. Moreover, annealing that contributes to the thickening of the crystals at expenses of the crystalline-amorphous interface may affect gas transport in semi-crystalline rubbery films.

Earlier work carried out on gas transport through coextruded linear low-density polyethylene LLDPE films revealed some unexpected facts [1,2]. For example, the increase in permeability observed in co-extruded LLDPE films after annealing is not due to changes in the diffusion coefficient (in fact this parameter slightly diminishes with annealing), but to a significant increase in the apparent solubility coefficient. On the other hand, the permeability coefficient of the films sharply increases at rather low upstream pressures [3], dependent on temperature and the nature of the penetrant.

A goal of this work is to investigate the transport of  $CO<sub>2</sub>$ in LDDPE films obtained by compression in order to learn whether co-extrusion is responsible for the differences observed in gas transport in these membranes when compared with gas permeation in rubbery membranes. For this purpose the permeation characteristics of  $CO<sub>2</sub>$  in LLDPE films were measured under different upstream pressures.

On the other hand, in the determination of the diffusion coefficient from permeation measurements, the concentration of the penetrant is not homogenous across the sample. NMR techniques may provide the means to measure the diffusion coefficient at constant concentration. By using this technique, Cain et al. [4,5] found that the translational diffusion coefficient of  $CO<sub>2</sub>$  in poly(dimethyl siloxane) has a wide distribution. Analysis of these results led these authors to conclude that the fraction of molecules of  $CO<sub>2</sub>$  compatible with the diffusion coefficient obtained from permeation measurements is rather small. In this work the diffusion coefficient of  $CO<sub>2</sub>$  in LLDPE films was also measured using NMR techniques with the aim of investigating how

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**Thermostated Bath** 

Fig. 1. Set-up for gas transport measurements.

the value of  $D$  thus obtained compares with that determined from permeation measurements.

#### 2. Experimental part

### 2.1. Characteristics of the LLDPE films

Pellets of a copolymer of 1-octene-co-ethylene (Dow LLDPE,  $\rho = 0.902 \text{ g cm}^{-3}$ , with roughly 8% content of 1-octene, were pressed between two hot plates to obtain films of  $276 \pm 7$  µm. The thermograms of the films, determined with a DSC-7 Perkin-Elmer calorimeter at a heating rate of  $20^{\circ}$ C min<sup>-1</sup>, present a melting endotherm that extends from 47 to  $110^{\circ}$ C with the maximum of the peak



Fig. 2. Illustrative plot showing the evolution with time of the pressure of CO<sub>2</sub> in the downstream chamber at  $p = 676.4$  cm of Hg and  $T = 25^{\circ}$ C.

located at  $99^{\circ}$ C. The degree of crystallinity of the film, obtained from the melting endotherms by assuming that the melting enthalpy is 290 J  $g^{-1}$  was 0.26.

### 2.2. Permeation measurements

The permeation of  $CO<sub>2</sub>$  in LLDPE films was measured in the experimental device shown in Fig. 1. Keeping valves 3, 4, 5 and 6 open and valve 2 closed, high vacuum  $(\sim 10^{-4}$  mbar) was made for 48 h in both the upstream and downstream chambers separated by the film. Then valves 4 and 5 were closed, valve 2 was opened and the  $CO<sub>2</sub>$  kept in a ballast bottle placed inside the thermostated bath, at a pressure close to that used in the experimental, flowed into the high pressure chamber. Taking zero as the time at which valve 2 was opened, the variation of the pressure with time in the downstream chamber was detected with a pressure sensor. Before each experiment, the leak curve was determined by measuring the variation of the pressure with time in the downstream chamber in vacuum.

## 2.3. NMR techniques

The dynamics of  $CO<sub>2</sub>$  in LLDPE was studied in a Bruker MSL-300 spectrometer using the spin echo technique. After removing the air in a tube containing small pieces of pellets of LLDPE inside,  $CO<sub>2</sub>$  at 9 atm of pressure was let in. The tube was closed in situ and the experiment was performed at  $25^{\circ}$ C.

To identify the relation between the gradient in Gauss  $cm^{-1}$  and the value that the set-up shows, the apparatus was previously calibrated by measuring the diffusion coefficient of a liquid of known diffusion coefficient, such as benzene  $(D = 2.21 \times 10^{-5} \text{ cm}^2 \text{ s}^{-1})$ .



Fig. 3. Variation of the permeability coefficient of carbon dioxide with the pressure of the upstream chamber at several temperatures.

## 2.4. Permeation results

An illustrative curve depicting the evolution with time of the pressure,  $p$ , in the downstream chamber is shown in Fig. 2. The curve presents at short times a transitory followed at longer times by a steady state region in which  $p$  is a linear function of  $t$ . The permeation curve in the whole time domain can be obtained by integrating Fick's second law using the appropriate boundary conditions. In the steady state region, the amount of gas  $Q(t)$  diffusing at time t is given by  $[6,7]$ 

$$
Q(t) = \frac{DC_{\rm h}}{l} \left( t - \frac{l^2}{6D} \right) \tag{1}
$$

where  $C_h$  is the gas concentration in the high pressure



Fig. 4. Variation of the diffusion coefficient of carbon dioxide with temperature for different values of the upstream pressure.

chamber,  $l$  the film thickness and  $D$  is the apparent diffusion coefficient. In steady state conditions, the flow of gas through a membrane can be written as

$$
J = -P\frac{p_1 - p_h}{l} \cong P\frac{p_h}{l} \tag{2}
$$

where P is the permeability coefficient and  $p_h$  and  $p_l$  are, respectively, the pressures of the gas in the upstream and downstream chambers. From Eq. (2) the following expression is obtained for the permeability coefficient:

$$
P = \frac{273}{76} \left( \frac{Vl}{ATp_h} \right) \left( \frac{dp}{dt} \right)
$$
 (3)

where  $V$  is the volume of the low pressure chamber,  $A$  the effective film area,  $p<sub>h</sub>$  the pressure of the gas in the high pressure chamber,  $p$  the pressure of the low pressure chamber and T is the absolute temperature of the thermostat. The unit used to express the permeability coefficient is the Barrer (= $10^{-10}$  cm<sup>3</sup> (STP) cm cm<sup>-2</sup> s cm of Hg).

The diffusion coefficient can be obtained from  $O(t)$  by the lag method suggested by Barrer [8]

$$
D = \frac{l^2}{6\theta} \tag{4}
$$

where  $\theta$ , the time lag, can be determined from the intercept of  $p(t)$  on the time axis.

Values of the permeability coefficient for  $CO<sub>2</sub>$  are plotted as a function of the upstream pressure, at different temperatures, in Fig. 3. It can be seen that at 50 and  $60^{\circ}$  C, the values of the permeability coefficient undergo a sharp increase with decreasing  $p<sub>h</sub>$ , in the low-pressure region. This upturn is not detected in the isotherms corresponding to  $25$  and  $40^{\circ}$ C in the range of pressures studied.



Fig. 5. Isotherms showing the variation of the solubility coefficient of carbon dioxide with the up-stream pressure at the temperatures indicated.

Results for the apparent diffusion coefficient of  $CO<sub>2</sub>$  for different upstream pressures are plotted as a function of temperature in Fig. 4. The curves show that the diffusion coefficient increases with increasing temperature and, in most cases, D tends to augment as the upstream pressure goes up.

The apparent solubility coefficient,  $S$ , is usually defined as

$$
S = P/D \tag{5}
$$

Values of the apparent solubility coefficient calculated from Eq. (5) are plotted as a function of the upstream pressure, at 40, 50 and  $60^{\circ}$ C, in Fig. 5. It can be seen that the resulting curves, at each temperature of interest, exhibit the same pattern as that shown by the permeability coefficient. Thus S decreases as  $p<sub>h</sub>$  increases, and in the region of low upstream pressures the solubility coefficient experiences a sharp increase with decreasing values of  $p<sub>h</sub>$ . It is worth noting that the increase occurring in the values of S is larger with increasing temperature.



Fig. 7. Plot of ln  $A(G)$  versus  $G^2$  for system CO<sub>2</sub>/polyethylene at 25<sup>o</sup>C and 676.4 cm Hg of pressure.

#### 2.5. NMR results

The sorbed molecules of  $CO<sub>2</sub>$  in the polymer matrix continuously change their spatial locations. In an inhomogeneous magnetic field, the component of the displacement in the direction of the field gradient changes the Larmor frequency of nuclear spins attached to the diffusing molecules. The amplitude of the spin echo is reduced as the molecules diffuse to greater distances during the interval  $\tau$ between the pulses. The spin echo method is based on the difference between the amplitude of the echo with and without a field gradient G applied for a time  $\delta$ . With the pulse sequence [9,10] shown in Fig. 6 the spin echo amplitude in the presence of a gradient  $A(G)$  is given by

$$
A(G) = A(0) \exp[-(\gamma G \delta)^2 D(\Delta - \delta/3)] \tag{6}
$$

where  $A(0)$  is the amplitude without gradient,  $\gamma$  the gyromagnetic ratio of the nucleus,  $\Delta$  the diffusion time, G the gradient (Gauss cm<sup>-1</sup>),  $\delta$  the time that the gradient is applied and D is the diffusion coefficient. The values of  $\delta$ and  $\Delta$  used in the spin echo sequence were  $5 \times 10^{-3}$  and  $15 \times 10^{-3}$  s. The correspondence between G in Gauss cm<sup>-1</sup> and G in arbitrary units was obtained by calibration with



Fig. 6. Pulsed-gradient spin echo scheme.

Table 1 Values of the individual parameters for  $CO<sub>2</sub>$  associated with the first and second modes of transport

$T({}^{\circ}C)$	b (cm Hg) <sup><math>-1</math></sup>	$10^3 \times k_D$ cm <sup>3</sup> of gas $(STP)/(cm3$ of polymer cm Hg)	$C'_{\rm H}$ cm <sup>3</sup> gas(STP)/ $cm3$ of polymer	
40	0.03	2.2	0.23	
50	0.08	2.1	0.18	
60	0.45	2.1	0.12	

benzene, using the value of  $2.21 \times 10^{-5}$  cm<sup>2</sup> s<sup>-1</sup> reported for the diffusion coefficient of this compound.

Values of  $A(G)$  for  $CO<sub>2</sub>$  in LLDPE were obtained for different values of G, at 9 atm of pressure. The plot  $ln A(G)$  against G, shown in Fig. 7, was fitted to a straight line from whose slope the value of  $5.65 \times 10^{-6}$  cm<sup>2</sup> s<sup>-1</sup> was obtained for the diffusion coefficient of  $CO<sub>2</sub>$ .

## 3. Discussion

The solubility coefficient of gases in rubbery films can be predicted by using the Flory–Huggins equation that accounts for the mixture of the gas in the liquid form with the polymer  $[11]$ . The value of S thus obtained is given by [12]

$$
\ln S = -\ln \overline{V}_{A} - (1 + \chi_{A}) + (1 + 2\chi_{A})\overline{V}_{A}C_{A}
$$

$$
- \frac{\lambda_{bA}}{RT_{bA}} \left(1 - \frac{T_{bA}}{T}\right)
$$

$$
= \ln k_{D} + bC_{A} \tag{7}
$$

where  $\overline{V}_{A}$  and  $C_{A}$  are, respectively, the partial molar volume and the concentration of the penetrant A,  $\lambda_{bA}$  and  $T_{bA}$  denote, respectively, the molar latent heat and the boiling point of A, and  $\chi_A$  is the enthalpic interaction parameter between the penetrant and the polymer. Eq. (7) reduces to Henry's equation at low concentrations and deviates positively from it at high concentrations. According to Eq. (7), the more condensable a gas is, the more soluble the gas in a rubbery membrane is. This prediction, however, is not borne out by the results of Fig. 5 where it can be seen that the solubility coefficient of  $CO<sub>2</sub>$  in LLDPE films decreases with increasing pressure.

On the other hand, the diffusion process depends on the interaction coefficient  $\zeta$  between the penetrant and the polymer matrix. By assuming that  $\zeta$  is governed by the fractional free volume,  $v_f$ , the thermodynamic diffusion coefficient,  $D_T$ , is given by [13]

$$
D_{\rm T} = \frac{\rm R}{} \frac{\zeta}{\zeta} = \rm R}{} \gamma A_{\rm D} \exp \left( -\frac{B_{\rm D}}{\phi_{\rm p} v_{\rm f}} \right) \tag{8}
$$

where  $\phi_p$  is the volume fraction of amorphous polymer, and

 $B<sub>D</sub>$  and  $A<sub>D</sub>$  depend on penetrant molecular size. A rather good approximation to the free volume is obtained from  $v_f =$  $[v_a(T) - v_c(T)]/v_a(T)$  where  $v_a$  and  $v_c$  are, respectively, the specific volumes of the amorphous and crystalline phases of LLDPE. Since the free volume is affected by pressure, temperature and amount of absorbed penetrant, the fractional free volume with respect to a reference state can be written as  $[14-17]$ 

$$
v_{f}(p, T, \phi_{A}) = v_{f}(p_{r}, T_{r}, 0) + \alpha_{f}(T - T_{r}) - \beta(p - p_{r}) + \gamma \phi_{A}
$$
\n(9)

where the subindex r denotes the reference state and  $\phi_A$  is the volume fraction of penetrant at the state defined by  $p$  and  $T$ . The parameters  $\alpha_f = (\partial v_f/\partial T)_r$ ,  $\beta = -(\partial v_f/\partial p)_r$  and  $\gamma =$  $(\partial v_f/\partial \phi_A)_r$  are positive constants that characterize the effectiveness of temperature, pressure and penetrant concentration for changing the free volume. The fractional free volume at constant temperature is the result of two at first sight opposing effects: the decrease of free volume, due to the hydrostatic pressure, and the increase in the value of  $v_f$ , caused by the sorption of the penetrant in the membrane. The results for the diffusion coefficient of  $CO<sub>2</sub>$ , shown in Fig. 4, are not totally conclusive with respect to which of the two effects is dominant. It seems, however, that at high temperatures the sorption effect overcomes the compression effect in the diffusion of  $CO<sub>2</sub>$  in LLDPE membranes.

The rather sharp decrease in the apparent solubility coef ficient of  $CO_2$  observed at 50 and 60°C in Fig. 5, in the low pressures region, can be explained by means of the dual mode model. According to the model, the apparent solubility of  $CO<sub>2</sub>$  in LLDPE could be the result of Henry's solution, and trapped molecules in holes or microvoids located in crystal defects and/or crystalline-amorphous regions' interphases. Accordingly, the solubility coefficient can be written as [18,19]

$$
S = k_{\rm D} + \frac{bC_{\rm H}'}{1 + bp} \tag{10}
$$

where  $k_D$  is Henry's solution coefficient,  $C'_H$  the Langmuir sorption capacity and  $b$  is an affinity parameter characterizing the ratio of rate constants for sorption and desorption. These parameters can be obtained by plotting the results for S against  $1/(1 + bp)$  for different values of b until a straight line is obtained. From the slopes and the intercepts of the straight lines of these plots, the values of  $k_D$  and  $bC'_H$ , respectively, are evaluated. The results for these parameters at 40, 50 and  $60^{\circ}$ C are shown in Table 1.

According to the dual mode theory [18,19], the diffusion of  $CO<sub>2</sub>$  in LLDPE membranes may occur by jumps of the dissolved molecules into non-permanent holes resulting from the micro-Browning motions of the chains in the amorphous phase (first mode), and partial mobility of the trapped molecules (second model) in the permanent holes. The dualmode model leads to the following expression for the



Fig. 8. Plot showing the fitting of values of the permeability coefficient of  $CO<sub>2</sub>$  to Eq. (11).

permeability coefficient:

$$
P = k_{D}D_{D} + D_{H}\frac{bC'_{H}}{1 + bp}
$$
 (11)

where  $D_D$  and  $D_H$  are, respectively, the apparent diffusion coefficients for the sorbed molecules in the dissolved and trapped modes. The values of P at 40, 50 and  $60^{\circ}$ C, plotted as a function of  $1/(1 + bp)$  in Fig. 8, fit to straight lines and the slopes and intercepts of which give, respectively,  $k_D D_D$ and  $bD_{\rm H}C'_{\rm H}$ . These values, in conjunction with those corresponding to  $k_D$  and  $bC_H'$  give  $D_D$  and  $D_H$ , the apparent diffusion coefficients for the sorbed molecules in the first and second modes, respectively. For comparative purposes, the values of the parameters of the dual mode model obtained in this work, together with those reported earlier for co-extruded LLDPE films  $[1-3]$ , are shown in Table 2. An inspection of these results shows that the contributions of the first  $(k_D D_D)$  and second  $(C'_H D_H)$  modes to the perme-

Table 2

Parameters of dual mode for CO<sub>2</sub> in co-extruded (LLDPE<sub>co-ext</sub>) and pressed  $(LLDPE<sub>press</sub>)$  films

Parameters of dual mode	$LLDPEco-ext$		LLDPE <sub>press</sub>	
	$T = 45^{\circ}C$	$40^{\circ}$ C	$50^{\circ}$ C	$60^{\circ}$ C
b (cm Hg) <sup><math>-1</math></sup>	0.06	0.03	0.08	0.45
$10^3 \times k_D$ cm <sup>3</sup> of gas $(STP)/(cm3$ of polymer	7.3	2.2	2.1	2.1
$cm Hg$ )				
$C'_{\rm H}$ cm <sup>3</sup> gas(STP)/cm <sup>3</sup> of polymer	0.53	0.23	0.18	0.12
$10^{10} \times k_D D_D$ (Barrers)	45.4	33.1	44.8	65.2
$10^{10} \times C_H' D_H$ cm <sup>3</sup> of gas $(STP)$ cm/(cm <sup>2</sup> s)	600	358	730	523
$10^7 \times D_D$ (cm <sup>2</sup> /s)	6.2	15.1	21.7	31.2
$10^7 \times D_{\rm H}$ (cm <sup>2</sup> /s)	1.1	1.6	4.1	4.3

ability coefficient of  $CO<sub>2</sub>$  through polyethylene films as well as the affinity parameter  $(b)$  are of the same order for the co-extruded and pressed LLDPE films. More important differences are observed in the values of  $k_D$  and  $D_D$  which for co-extruded LLDPE are, respectively, one half and twice the respective values of these quantities for pressed LLDPE.

The co-extruded LLDPE films used in previous works  $[1-\frac{1}{2}]$ 3] were also 1-octene-co-ethylene copolymers with roughly 8% mol content of the first comonomer. The films were made of three layers, i.e.  $C(15 \text{ wt}\%)A(70 \text{ wt}\%)B(15 \text{ wt}\%)$ . A and C are Dowlex 2247 ( $\rho = 0.902$  g cm<sup>-3</sup>) like that used in this study, and layer B is Dowlex 2291 ( $\rho = 0.912$  g cm<sup>-3</sup>). The similarity of the transport of  $CO<sub>2</sub>$  in co-extruded and molded LLDPE films suggests that the microstructure developed in the films by the co-extrusion process, which so strongly affects the mechanical properties of LLDPE, hardly affects the transport of carbon dioxide through this material.

Whereas the diffusion coefficient obtained from permeation measurements is an average quantity given by

$$
D = \frac{1}{C_{\rm h} - C_1} \int_{C_1}^{C_{\rm h}} D(C) \, dC \tag{12}
$$

where  $C_h$  and  $C_l$  are, respectively, the concentration of the gas at the high and low pressure sides of the membrane, the concentration is kept constant throughout the sample in the determination of  $D$  using the NMR technique. The value obtained for the diffusion coefficient of  $CO<sub>2</sub>$  by permeation experiments under an upstream pressure of 9 atm is  $6.86 \times 10^{-7}$  cm<sup>2</sup> s<sup>-1</sup>. This quantity is nearly one order of magnitude lower than the value of  $5.65 \times$  $10^{-6}$  cm<sup>2</sup> s<sup>-1</sup> found by NMR under a hydrostatic pressure of nearly the same magnitude. In principle, this discrepancy could be attributed to differences in morphology in the samples used, pellets in NMR and films in permeation. However, the fact that degree of crystallinity of the pellets and films was similar, specifically,  $29$  and  $26\%$ , respectively, seems to rule out this possibility. The explanation of the difference in the values of  $D$  obtained by the two techniques may lie in that NMR is an equilibrium measurement of self-diffusion whereas the permeation measurement is not an equilibrium measurement.

Earlier studies carried out on the dynamics of  $CO<sub>2</sub>$  in poly(dimethylsiloxane) rubber by NMR showed that the translational diffusion has a wide distribution [4,5]. From the spin-lattice and spin-spin relaxation times as well as nuclear Overhauser determined at three magnetic fields as a function of temperature, the diffusion coefficient of  $CO<sub>2</sub>$  in the silicone membrane was calculated finding that this parameter has a wide distribution. In fact, the values of the selfdiffusion coefficient seem to span a range of nearly four orders of magnitude. Therefore, the value of D obtained in this work from NMR could be very well an average corresponding to the distribution of the diffusion coefficient of  $CO<sub>2</sub>$  in LLDPE.

As for the permeation measurements, the expression that

relates the permeability coefficient to the diffusion coefficient may not be a simple one. There are boundary effects [10] in the diffusion and non-linear concentration gradients in the sample that have to be accounted for. Moreover, in semi-crystalline membranes permeation will be limited only to the non-crystalline regions and the concentration will have to be scaled. All the effects involved in the determination of the diffusion coefficient by NMR and permeation measurements are difficult to analyze quantitatively. In most cases, however, trends between the diffusion coefficients obtained by NMR and permeation are similar, and one order of magnitude difference in the numbers is not unusual.

## 4. Conclusions

Transport of  $CO<sub>2</sub>$  in LLDPE films obtained by co-extrusion and compression experiences a high increase at moderately high temperatures as the upstream pressure  $p<sub>h</sub>$ decreases, in the region of low values of  $p<sub>h</sub>$ . This behavior is described by the dual-mode model finding that most of the parameters of the model are similar for both types of films.

The value of the diffusion coefficient of  $CO<sub>2</sub>$  in compressed films, determined by  ${}^{13}C$  NMR, is nearly one order of magnitude higher than that obtained from permeation measurements. This discrepancy seems to be too high to be attributed to the fact that the NMR measurement is an equilibrium measurement and the permeation is not. Other causes discussed in the text need to be investigated to explain the discrepancy observed.

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